

# Theoretical and Experimental Evidence of Nonreciprocal Effects on Magnetostatic Forward Volume Wave Resonators

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**Abstract**—An unexpected nonreciprocal behavior is observed on Magnetostatic Forward Volume Waves (MSFVW) Straight Edge Resonators (SER's) using YIG films. The nonreciprocal effect is theoretically predicted by taking into account the demagnetizing effect of the dc biasing magnetic field when calculating the forward and reverse propagation constants of MSFVW inside the YIG film. Hence, a nonreciprocal scattering matrix of the MSFVW-YIG-SER coupled to its transducers is calculated. The measured values of the transmission coefficients agree extremely well with the predicted ones. They both present an important nonreciprocal phase behavior.

## I. INTRODUCTION

MAGNETOSTATIC waves planar devices including YIG films operate as frequency-tunable delay-lines or resonators in frequency synthesizers, channel-filters and tuned oscillators. The models found in the literature are based for both structures on the characterization of the propagation mechanism of magnetostatic waves in a planar YIG film which is coupled to planar transducers on a substrate. Assuming a propagation of MSW in the  $z$ -direction of Fig. 1, three implicit dispersion relations are obtained when the film is considered as infinite in both the  $x$ - and  $z$ -directions, depending on the orientation of the uniform dc biasing magnetic field  $H_0$  [1], [2]. Forward Volume MSW occur when  $H_0$  is  $y$ -oriented. When the YIG film has a finite width in the  $x$ -direction, O'Keeffe and Patterson [3] proposed to approximate its edges by perfect magnetic walls (PMW) at planes  $x = 0, W$ . W. Chang and all [4] have used this approximation for characterizing a two-port MSFVW-YIG-SER: they evaluated the resonant frequencies from the propagation constant obtained by modifying the MSFVW dispersion relation as in [3]. They, however, presented only magnitude measurements of their configuration, with no mention of its phase behavior.

To our knowledge, no attempt has been made to characterize both magnitude and phase of two-port YIG-SER's having no contact with the planar transducer. On the other hand, the propagation of MSFVW described by the simple dispersion relations mentioned above, and their coupling mechanism to transducers in case of delay lines, is reciprocal even

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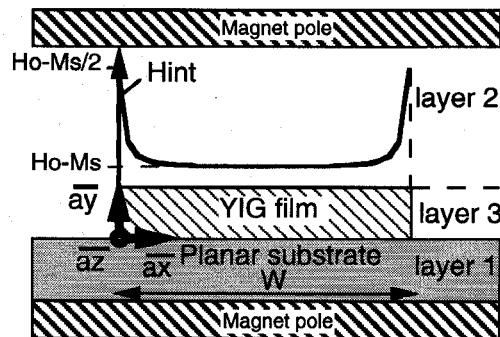


Fig. 1. Topology of a YIG film biased by a uniform external dc magnetic field  $H_0$  and profile of the internal dc field  $H_{\text{int}}$ .

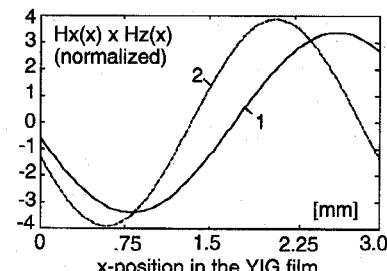


Fig. 2. Calculation of the  $x$ -dependence of the product  $H_x H_z$ . Curve 1:  $N_y(x)$  nonuniform, Curve 2:  $N_y(x)$  uniform.

when a width effect is considered, as stated by a number of authors [5], [1]. In this letter, we present a theoretical and experimental characterization of the scattering matrix of MSFVW-YIG-SER's coupled to microstrip transducers, pointing out a nonreciprocity in the phase behavior. This is shown to be due to the nonuniform demagnetizing effect induced by the finite width of the SER. Using a newly developed variational principle [6], based on an explicit second-order equation for the propagation constant, the nonuniformity of the internal dc-field is introduced in the permeability tensor of the YIG, while keeping simple trial fields.

## II. DESCRIPTION OF THE MODEL

A YIG film (Fig. 1) is biased by a uniform external dc magnetic field  $H_0$  along the  $y$  axis. The internal field  $H_{\text{int}}$  satisfies the boundary conditions at the interfaces between the ferrite and the air [7], via the demagnetizing factor  $N_y$

$$H_{\text{int}} = H_0 - N_y M_s. \quad (1)$$

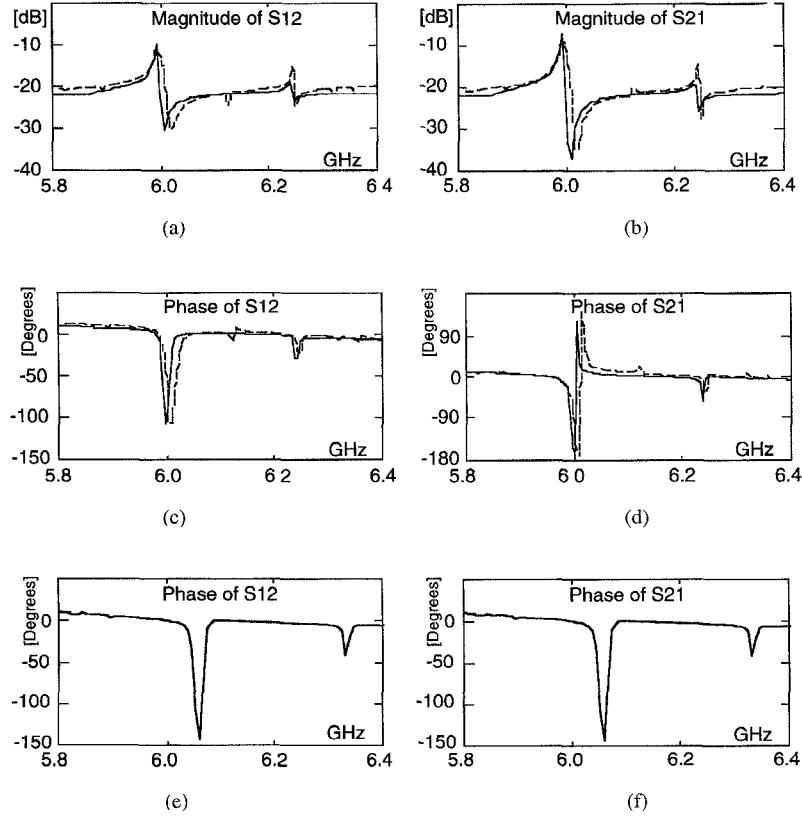


Fig. 3. Comparison of modeling (—) and measurement (---) of transmission coefficients of a MSFVW-YIG-SER (parameters of the YIG film: saturation magnetization = 1780 G, thickness = 0.1 mm, width = 2 mm, length = 1 mm, external dc magnetic field = 3750 Oe). (a) Magnitude of  $S_{12}(N_y(x))$  from [8]. (b) Magnitude of  $S_{21}(N_y(x))$  from [8]. (c) Phase of  $S_{12}(N_y(x))$  from [8]. (d) Phase of  $S_{21}(N_y(x))$  from [8]. (e) Phase of  $S_{12}(N_y(x) = 1)$ . (f) Phase of  $S_{21}(N_y(x) = 1)$ .

We use the first-order term of the solution proposed by Joseph and Schlomann [8, (16), (17)] who develop  $N_y$  as a series of ascending powers in  $(H_0/M_s)$ . The resulting  $H_{\text{int}}$  is nonuniform over the sample-width (Fig. 1), hence the permeability tensor in the YIG film varies with  $x$ .

Maxwell's equations in the magnetostatic range for the three media of Fig. 1 are used as described in [1]. Using a variational principle [6], suitable approximations for the trial magnetostatic potential are

$$\Phi^{\text{YIG}}(x, y, z) = [X_1 \sin k_x x + X_2 \cos k_x x] Y_i(y) e^{-\gamma z} \quad \text{for } 0 \leq x \leq W \quad (2a)$$

$$\Phi^{\text{L}}(x, y, z) = L^i e^{+\gamma_n x} Y_i(y) e^{-\gamma z} \quad \text{for } x \leq 0 \quad (2b)$$

$$\Phi^{\text{R}}(x, y, z) = R^i e^{-\gamma_n (x-W)} Y_i(y) e^{-\gamma z} \quad \text{for } x \geq W \quad (2c)$$

with  $k_x$  the wavenumber along the  $x$ -axis and

$$\gamma_n = \sqrt{[-\mu_{11,i} k_x^2 + (\mu_{11,i} - 1)\gamma^2 - k_z^2]}.$$

Because of the nonuniformity in the demagnetizing effect, the relative permeability tensor  $\bar{\mu}_3$  is spatially nonuniform in the YIG film ( $0 \leq x \leq W$ ), while the permeability tensor is isotropic in the layers 1 and 2. The magnetic field in each area is obtained as the gradient of the scalar potential whose  $y$ -dependences  $Y_i(y)$  in each layer are similar to those found in [1].

The  $x$ -dependence of the field has been kept unknown in expressions (2a) in order to use a more rigorous boundary condition than the PMW-condition. On the left and right sides of the YIG film in layer 3 the potential is assumed to exponentially decrease away from the edges (see (2b) and (2c)). Imposing at the planes  $x = 0$  and  $x = W$  of layer 3 the continuity between the  $H_y$ ,  $H_z$  and  $B_x$  components outside and inside the YIG film, we obtain

$$\tan k_x W = \frac{-2\gamma_n \mu_{11,3} k_x}{\gamma_n^2 - (\mu_{21,3}\gamma)^2 - (\mu_{11,3} k_x)^2} \quad (3)$$

$$X_2 = [(\mu_{11,3} k_x)/(\gamma \mu_{21,3} + \gamma_n)] X_1. \quad (4)$$

Curve 1 of Fig. 2 shows the resulting  $x$ -dependence of the product of the  $x$ - and  $z$ -components of the magnetic field when solving (3) and (4) with the nonuniform  $H_{\text{int}}$ . Curve 2 corresponds to the uniform case  $N_y(x) = 1$ . Since our formulation of the propagation constant  $\gamma$  involves integrals of second powers of fields over the cross-section of Fig. 1, any nonreciprocal effect will be induced by a contribution of the product  $H_x H_z$ . The sign of this product only varies indeed with  $\gamma$ . When  $H_{\text{int}}$  is nonuniform, the total contribution of this term over the whole  $x$  range does not vanish, because it is affected by nonequal contributions of the permeability tensor. On the other hand, when the permeability tensor is uniform, the compensation holds because the product  $H_x H_z$  varies periodically over the  $x$ -range and is affected by a constant per-

meability. So its total contribution is zero. As a consequence, when  $H_{\text{int}}$  is nonuniform, we obtain two different complex propagation constants depending respectively on the positive or negative propagation  $z$ -direction. When  $H_{\text{int}}$  is uniform, the forward and reverse propagation constants are identical.

Hence, we model a two-port MSFVW-YIG-SER as a transmission line having different forward and reverse propagation constants. It is connected to the microstrip transducers via load impedances which are calculated as for dielectric resonators [9] using the energy contained in the YIG-SER [10]. The final result is shown in Fig. 3(a)–(d) where the modeled forward and reverse transmission scattering terms (solid lines) are compared to the measured ones (dashed lines). The agreement is excellent. The difference between the forward [Fig. 3(a)] and reverse [Fig. 3(b)] absorption peaks is correctly predicted. A strong nonreciprocity is observed in both modeling and experiment: the forward phase [Fig. 3(c)] exhibits a variation of  $100^\circ$  at the main resonance, while the reverse one [Fig. 3(d)] undergoes a sudden drop of  $360^\circ$ . When the demagnetizing effect is taken uniform in the model ( $N_y(x) = 1$ ), the nonreciprocal effects disappear in the simulation, as can be seen for the phases, represented in Fig. 3(e) and (f). Similar results have been obtained at  $X$ -band, with measurements confirming the model.

### III. CONCLUSION

We have pointed out a nonreciprocal behavior of MFVW-YIG-SER's, induced by the nonuniformity of the demagnetization over the finite width of the resonator. Simple trial fields are derived which, combined with the new variational formulation

we recently developed, offer an efficient way to predict the phase and magnitude behavior of the scattering terms of the SER coupled to its transducers. The excellent agreement observed between the simulation and the experiment validates the theoretical formulation used to take into account the demagnetizing effect in a YIG-SER.

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